

# Visco-elastic modelling for asphalt pavements – Software ViscoRoute

A. Chabot, P. Tamagny & D. Poché

*Laboratoire Central des Ponts et Chaussées, Division Matériaux et Structures de Chaussées, Nantes, France.*

D. Duhamel

*Ecole Nationale des Ponts et Chaussées, Laboratoire d'Analyse des Matériaux et Identification, Marne la Vallée, France.*

**ABSTRACT:** Huet-Sayegh model (1963) gives a set of constitutive equations of a visco-elastic material which accounts well for the behaviour of asphalt pavement layers, especially regarding thermal effects. This model allows rather good predictions of experimental data.

The French pavement design method consists in a pavement mechanistic analysis based on the Burmister multilayer elastic model (1943) -LCPC software ALIZE (1982)-. In that model, the Huet-Sayegh behaviour is taken into account with its equivalent elastic modulus at the 15°C French average temperature and a 10 Hz frequency. That frequency value is assumed to be equivalent to the standard 72 km/h French vehicle speed. Such semi-analytical calculations provide relatively good stress and strain fields for heavy traffic pavements but it is less satisfactory for flexible pavements with low traffic, for high temperature gradients and for the analysis of damages under slow heavy loads. Therefore the complete visco-elastic behaviour of each asphalt pavement layers has to be taken into account. The aim of this paper is to present a thermo-visco-elastic multi-layer model using the Huet-Sayegh behaviour. By means of the Fast Fourier transform method, the equations of the model are solved in the coordinate system of the moving load. Results are successfully compared with an analytical solution, finite element results and accelerated pavement testing data. A software called ViscoRoute (i.e. ViscoRoad) based on this modelling has been developed. The second part of this work deals with the relevance of the assumption on the time-frequency equivalence of the French design method.

**KEY WORDS:** Modelling, mechanics, asphalt pavement, visco-elasticity, software

## 1 INTRODUCTION

For asphalt mixes, the visco-elastic model proposed initially by Huet (1963) and Sayegh (1963) predicts very accurately laboratory complex modulus test results (de la Roche, 1996). The French design method uses the Burmister multilayer elastic model (1943). In that model, each layer has an elastic behaviour. The Huet-Sayegh behaviour is taken into account with its equivalent elastic modulus at the 15°C French average temperature and a 10 Hz frequency. That frequency value is assumed to be equivalent to the standard 72 km/h French vehicle speed. This model is implemented in numerous software like ALIZE from LCPC described in (Autret, 1982). It gives by semi-analytical calculations relatively good stress and strain fields for heavy traffic pavements

specially for base course made with classical materials. On the contrary, for flexible pavement with low traffic, for high temperature gradients and for the analysis of damages under slow heavy loads, it is necessary to take into account the visco-elastic behaviour of asphalt materials.

In this paper, a model for a semi-infinite multilayer structure taking into account the thermo-visco-elastic Huet-Sayegh constitutive relation is presented. The comparison on results of this model with analytical solutions and with finite element results from the module CVCR (visco-elastic calculation under moving load in French) (Heck *et al.* 1998) of the finite element code CESAR-LCPC on accelerated pavement testing data are successfully given. The main interest of this method compared to the two others is the very short computational time for a multilayer structure. A software, called ViscoRoute (French name for viscoroad) based on this modelling has been developed. The second part of this paper deals with the relevance of the assumption on the time-frequency equivalence of the French design method.

## 2 MECHANICAL MODEL

The proposed method consists in using the asphalt mixes Huet-Sayegh behaviour and a double Fourier transform to solve analytically the problem along the depth in the space of horizontal wavenumbers. The solution in the moving load reference system is got by a double inverse Fourier transform. Then one can calculate the displacements and the stresses in different positions in the road according to the intensity of the load, the temperature or the speed of the vehicles.

### 2.1 Visco-elastic constitutive relation

The road structure is a multilayer half-space made of elastic or visco-elastic layers with a thermovisco-elastic behaviour described by the Huet and Sayegh (1963) constitutive relation. This model consists in two parallel branches. The first is made of a spring and two parabolic dampers giving the instantaneous and retarded elasticity of asphalt. The second is made of a spring leading to the static or long-term elasticity of asphalt (Figure 1).

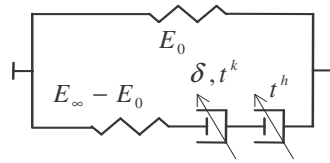


Figure 1: Scheme of the Huet-Sayegh rheological model.

The parameter  $E_\infty$  is the instantaneous elastic modulus,  $E_0$  the static elastic modulus,  $k$  and  $h$  are the exponents of the parabolic dampers ( $1 > h > k > 0$ ) and  $\delta$  is a positive adimensional coefficient balancing the contribution of the first damper in the global behaviour. The visco-elastic behaviour, at the frequency  $\omega$  and the temperature  $\theta$ , is given by the complex modulus:

$$E^*(\omega, \theta) = E_0 + \frac{E_\infty - E_0}{1 + \delta(j\omega\tau(\theta))^{-k} + (j\omega\tau(\theta))^{-h}} \quad (1)$$

with  $e^{j\omega t}$  the time variation,  $\tau(\theta) = \exp(A_0 + A_1\theta + A_2\theta^2)$  a function of the temperature depending on three scalar parameters  $A_0$ ,  $A_1$  and  $A_2$ . In the following the temperature is considered as uniform and constant and the dependency on  $\theta$  of the different variables will be omitted.

## 2.2 Moving load

The load is a constant uniform pressure on the rectangle  $[-a,a] \times [-b,b]$ , with the total strength  $f_0 = 4abp_0$ , and moving at the constant velocity  $V$  (Figure 2).

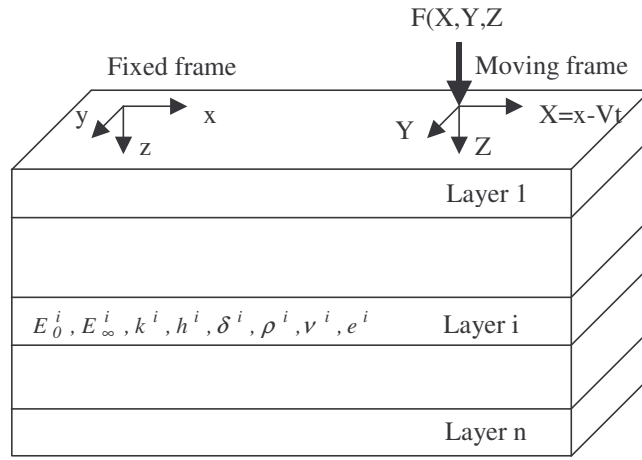


Figure 2: Moving load on a multilayer half-space with the Huet and Sayegh model

In the fixed coordinate system, the equilibrium equation for each layer  $i$  is given by:

$$\text{Div}(\underline{\underline{\sigma}}(x, y, z, t)) = \rho_i \frac{\partial^2 u(x, y, z, t)}{\partial t^2} \quad (2)$$

Where  $u(x, y, z, t)$  is the displacement vector,  $\rho_i$  the density of the material in the layer  $i$  and  $\underline{\underline{\sigma}}(x, y, z, t)$  the Cauchy stress tensor.

The boundary conditions are:

- On the free surface:  $\underline{\underline{\sigma}}(z=0) \cdot n = -p_0$  on the rectangle  $[-a,a] \times [-b,b]$  and 0 elsewhere, where  $n$  is the exterior normal.
- At the interface between layers  $i$  and  $i+1$ :  $\underline{\underline{\sigma}}(x, y, z_i^+, t) \cdot n = \underline{\underline{\sigma}}(x, y, z_i^-, t) \cdot n$  and  $u(x, y, z_i^+, t) = u(x, y, z_i^-, t)$  (perfect sticking).
- At  $z = +\infty$ , the condition of vanishing fields,  $\underline{\underline{\sigma}}(x, y, +\infty, t) = 0$  and  $u(x, y, +\infty, t) = 0$ .

To pass in the moving coordinate system, one makes the substitution  $(x, y, z) \rightarrow (X - Vt, Y, Z)$ , which leads, for stationary regimes in which the load is independent from time, to the following relationship:

$$\text{Div}(\sigma(X, Y, Z)) = \rho_i V^2 \frac{\partial^2 u(X, Y, Z)}{\partial X^2} \quad (3)$$

To go further, we will take the Fourier transform of the precedent relation in the directions X and Y. The Fourier transform  $A^*$  of any tensor  $A$  is defined by:

$$A(X, Y, Z) = \frac{1}{4\pi^2} \int_{-\infty-\infty}^{+\infty+\infty} \int A^*(k_1, k_2, Z) e^{jk_1 X} e^{jk_2 Y} dk_1 dk_2 \quad (4)$$

It is then possible, for each layer  $i$ , to write the constitutive relation in the Fourier transform domain under a multiplicative form similar to the elastic relations in the Euclidian space:

$$\sigma^*(k_1, k_2, Z) = 2\mu_i^*(k_1 V) \varepsilon^*(k_1, k_2, Z) + \lambda_i^*(k_1 V) \text{tr}(\varepsilon^*(k_1, k_2, Z)) \mathbf{I} \quad (5)$$

where  $\lambda_i^*(k_1 V)$  and  $\mu_i^*(k_1 V)$  depend on the complex modulus  $E_i^*(k_1 V)$  of the layer  $i$  like the Lamé coefficient in the elastic case and  $\varepsilon^*$  is the strain tensor.

Putting relation (5) in (3), Nguyen (2002) gives, in the wavenumber domain, the equilibrium equation in each layer  $i$  as:

$$A_i \frac{\partial^2 U^*}{\partial Z^2} + jB_i \frac{\partial U^*}{\partial Z} - C_i U^* = 0 \quad (6)$$

with:

$$A_i = \begin{pmatrix} c_{si}^2 & 0 & 0 \\ 0 & c_{si}^2 & 0 \\ 0 & 0 & c_{pi}^2 \end{pmatrix} \quad B_i = \begin{pmatrix} 0 & 0 & k_1(c_{pi}^2 - c_{si}^2) \\ 0 & 0 & k_2(c_{pi}^2 - c_{si}^2) \\ k_1(c_{pi}^2 - c_{si}^2) & k_2(c_{pi}^2 - c_{si}^2) & 0 \end{pmatrix} \quad (7)$$

$$C_i = \begin{pmatrix} k_1^2(c_{pi}^2 - V^2) + k_2^2 c_{si}^2 & k_1 k_2 (c_{pi}^2 - c_{si}^2) & 0 \\ k_1 k_2 (c_{pi}^2 - c_{si}^2) & k_1^2(c_{si}^2 - V^2) + k_2^2 c_{pi}^2 & 0 \\ 0 & 0 & k_1^2(c_{si}^2 - V^2) + k_2^2 c_{si}^2 \end{pmatrix}$$

where  $c_{pi}$  and  $c_{si}$  are respectively the longitudinal and transverse complex velocities in the layer  $i$ . Looking for solutions of  $\mathbf{U}^*(k_1, k_2, Z)$  under an exponential form, the relation (6) applied in a layer leads to a solution which can be written as a sum of six eigenvectors. This gives a displacement field in the layer  $i$  which depends on 6 complex parameters, noted  $(\beta_{1i}^-, \beta_{1i}^+, \beta_{2i}^-, \beta_{2i}^+, \beta_{3i}^-, \beta_{3i}^+)$ , and can be written as:

$$\begin{aligned}
U_1^* &= k_1 \beta_{1i}^- e^{-\kappa_p Z} + \kappa_s \beta_{3i}^- e^{-\kappa_s Z} + k_1 \beta_{1i}^+ e^{\kappa_p Z} - \kappa_s \beta_{3i}^+ e^{\kappa_s Z} \\
U_2^* &= k_2 \beta_{1i}^- e^{-\kappa_p Z} + \kappa_s \beta_{2i}^- e^{-\kappa_s Z} + k_2 \beta_{1i}^+ e^{\kappa_p Z} - \kappa_s \beta_{2i}^+ e^{\kappa_s Z} \\
U_3^* &= j \kappa_p \beta_{1i}^- e^{-\kappa_p Z} + j k_2 \beta_{2i}^- e^{-\kappa_s Z} + j k_1 \beta_{3i}^- e^{-\kappa_s Z} - j \kappa_p \beta_{1i}^+ e^{\kappa_p Z} + j k_2 \beta_{2i}^+ e^{\kappa_s Z} + j k_1 \beta_{3i}^+ e^{\kappa_s Z}
\end{aligned} \tag{8}$$

$$\text{with } \kappa_p = \sqrt{\left(1 - \frac{V^2}{c_p^2}\right) k_1^2 + k_2^2} \quad \text{and} \quad \kappa_s = \sqrt{\left(1 - \frac{V^2}{c_s^2}\right) k_1^2 + k_2^2}$$

The displacement is a function of the horizontal wavenumbers  $k_1$  and  $k_2$  and of the depth  $Z$ . To avoid too large terms coming from the exponentials, the origin in each layer has been chosen at the top of the layer. Using relation (5) the stress tensor can also be written as a function of the six parameters  $(\beta_{1i}^-, \beta_{1i}^+, \beta_{2i}^-, \beta_{2i}^+, \beta_{3i}^-, \beta_{3i}^+)$ .

### 2.3 Multilayer Solution

The displacements and stresses at a point in the structure can also be written under a matrix form. Then, the continuity relationships for the displacements and the stresses are written at each interface between layers  $i$  and  $i+1$ . Under this form the transfer matrix between the two layer field involves only terms with negative exponentials avoiding a possible overflow in the calculations. One can go on the process and obtain a relation between layer  $i-1$  and  $i+1$  in a new transfer matrix. Continuing the process, a relation between the first and the last layers can be written.

In fact the condition at infinity and the load boundary condition on the free surface with:

$$\mathbf{f}(k_1, k_2) = \frac{4 \sin(k_1 a) \sin(k_2 b)}{k_1 k_2} \mathbf{f}_0 \tag{15}$$

where  $\mathbf{f}_0$  is a constant vector giving the direction of the load, usually normal (tangential loads could also be considered) allow to calculate the six parameters  $(\beta_{1i}^-, \beta_{1i}^+, \beta_{2i}^-, \beta_{2i}^+, \beta_{3i}^-, \beta_{3i}^+)$  in the first layer ( $i=0$ ) then in the other layers.

The displacements and stress fields in the wavenumber domain can so be determined. Then a double inverse Fourier transform allows getting the displacements and stresses in the real domain. More details on the calculation are given in (Nguyen, 2002) (Duhamel *et al.*, 2005). The main interest of this method is the very short computational time for a multilayer structure in comparison with finite element method (FEM).

Software, called ViscoRoute (French name for viscoroad), based on this modelling has been developed under Visual Basic ® environment in its first version.

## 3 VALIDATIONS

In a first step the results are compared with data obtained by other methods (analytical or finite elements). The loading is the same for every single loading case: a moving 65kN uniform load on

a squared surface with sides  $2a=2b=0.22158$  m (standard axle in the French road design method). Unless otherwise specified, the velocity of the load along the  $x$  axis is  $5 \text{ ms}^{-1}$ , the parameters of the  $\tau(\theta)$  function are  $A_0 = -0.342$ ,  $A_1 = -0.401$ ,  $A_2 = 0.002954$  and the temperature  $\theta$  is  $15^\circ \text{ C}$ .

### 3.1 Homogeneous Half-Space Case – Comparison with analytical solutions

The first case is an infinite homogeneous half-space with a visco-elastic material having a Poisson ratio of 0.3 and Young moduli  $E_0 = 70 \text{ Mpa}$  and  $E_\infty=29914 \text{ Mpa}$ . The dampers are supposed degenerated with:  $\delta = 0$ ,  $h=0$ . Consequently, the material is equivalent to an elastic material with modulus  $14992 \text{ Mpa}$  (Figure 3). The second case is for a material with similar properties except that the  $h$  coefficient is 0.3 (Figure 4). So this second case is a true visco-elastic material. The curves give the vertical displacements  $w$  along  $x$  for the depths  $z1= -0.0165$  m and  $z2= -0.2444$  m.

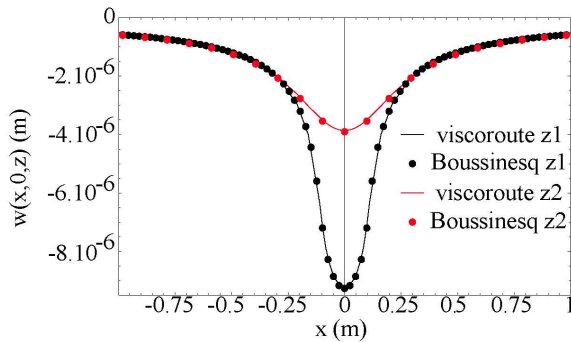


Figure 3: Elastic half-space

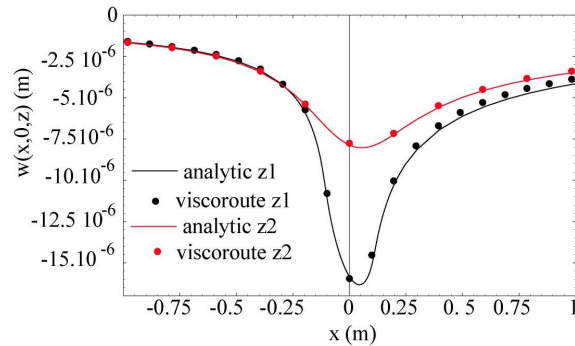


Figure 4: Visco-elastic half-space

They show that the comparisons with the analytical Boussinesq solution given by (Johnson, 1992) in the elastic case and the semi-analytical solution developed by (Chabot and Piau, 2001) in the visco-elastic case are very good (Duhamel *et al.*, 2005).

### 3.2 Multi-layered Case

The second comparison is shown in Figure 5. The structure is made of a virtual visco-elastic layer of depth 0.08 m ( $\nu = 0.5, E_0 = 0, E_\infty = 29914 \text{ Mpa}, h = 0.5, \delta = 0$ ) put on a 0.24 m elastic layer ( $\nu=0.35, E=10000 \text{ Mpa}$ ) and on an elastic soil massif ( $\nu=0.35, E=50 \text{ Mpa}$ ). The comparison is made with finite element results coming from the CVCR modulus of the Cesar-LCPC software. In the finite element calculation, a soil depth of 15 m is modelled and the displacement boundary conditions are applied at 10 m on both sides of the load. The symmetry of the problem relative to the  $xOz$  plane has been used. To get a precise comparison with the finite element results the depth of the soil has been limited to 15m in our model and a massif with a very high Young's modulus has been put under it, simulating a rigid boundary condition as for the finite elements calculation. The results are presented for the depth  $z3= -0.0165$  m and  $z4= -0.37$  m (Figure 5).

The comparison between the finite element results and our model is very good. It is to be noted that for a calculation with a full soil massif in our model instead of a 15m soil resting on a rigid massif, the comparison between the two results are not so good. It means in fact that an

insufficient number of elements has been used in the soil discretization by the finite element method and that a much larger depth than 15m should had been used.

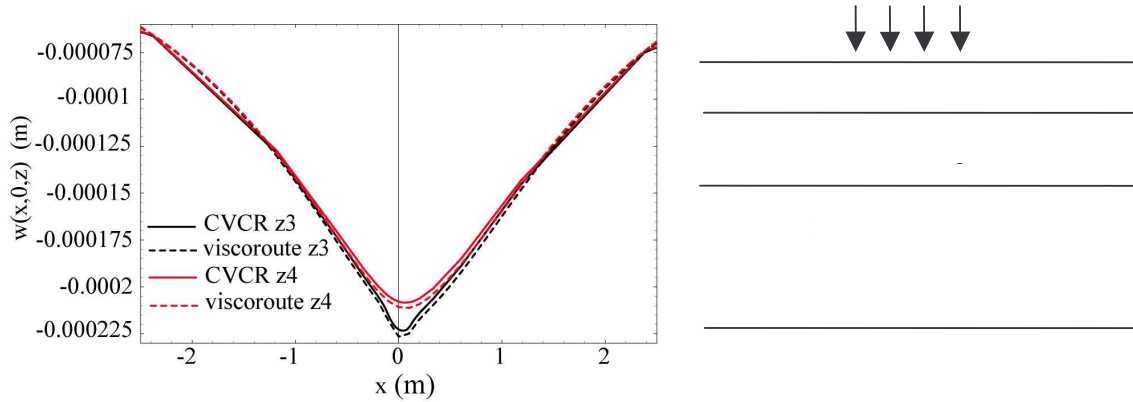


Figure 5: Comparison between CVCR FEM calculus and Viscoroute model

### 3.3 Multiple Loading Case

The last validation is made on accelerated airfield pavement experiment results in the case of multiple loading and material properties obtained with the help of inverse finite element calculus. More details can be found in the Vila report (Vila, 2001) and different publications on the A380 Pavement experimental program (PEP) as in (PetitJean *et al.*, 2002).

The pavement section illustrated here is supposed to be made of two identical visco-elastic bituminous layers (BB: asphalt concrete and GB: asphalt gravel) of depth 0.08m and 0.24m ( $\nu=0.35$ ,  $E_0=65\text{MPa}$ ,  $E_\infty=30000\text{Mpa}$ ,  $h=0.787$ ,  $k=0.25$ ,  $\delta=1.58$ ,  $A_0=3.597$ ,  $A_1=-0.382$ ,  $A_2=0.00179$ ) put on a 0.60 m elastic unbound granular material (GRH: humidify reconstituted crushed gravel) layer ( $\nu=0.35$ ,  $E=150\text{Mpa}$ ) and on a elastic soil ( $\nu=0.35$ ). The elastic soil is supposed to be composed of two reconstructed subgrade layers of depth 1m each ( $E_1=70\text{MPa}$ ;  $E_2=150\text{MPa}$ ) layered on a rigid subgrade ( $E=30000\text{MPa}$ ).

The simulation vehicle used is modular. One module figures out one bogie. It can be either a 4 to 6-wheels module or a basic diablo (2-wheels). Each module is individually loaded with applicable steel plates and ensure quasi-uniform supposed distribution (PetitJean *et al.*, 2002).

In the case illustrated here, the speed of the 4-wheels module bogie of the corresponding A340 aircraft is supposed to be 0.66m/s. Each load applies a 369.6kN uniform pressure on a rectangular surface ( $2a=0.56\text{m}$  and  $2b=0.40\text{m}$ ). Wheelbases are 1.98m along the longitudinal axe x and 1.40m along the transversal axe y (Figure 6). Inside bituminous layers, the thermal sensors give a thermal distribution such as : at the top of the pavement section there is 10.7°C, at 0.01m depth 10.2°C, at 0.08 and 0.20m depth 9.7°C and at 0.32m depth 9.3°C (Vila, 2001).

Being in the linear elastic mode, ViscoRoute calculus are made for one single load and then are superimposed for the four wheels by combination of the results on the free Scilab ® software.

To illustrate visco-elasticity effects, specially visible on multiple loading case, we choose here to show on Figure 6 (with soil mechanical convention of sign that means that compression is positive) the comparison between experimental results obtained by a strain transversal sensor

putted at the bottom of the Bituminous Gravel (GB), ViscoRoute result and a equivalent elastic result ( $T_{\text{average}} = 9.7^{\circ}\text{C}$ ,  $f = 0.33 \text{ Hz}$ ,  $E_{eq}^* = 11670.4\text{MPa}$  (1)) (Loft, 2005).

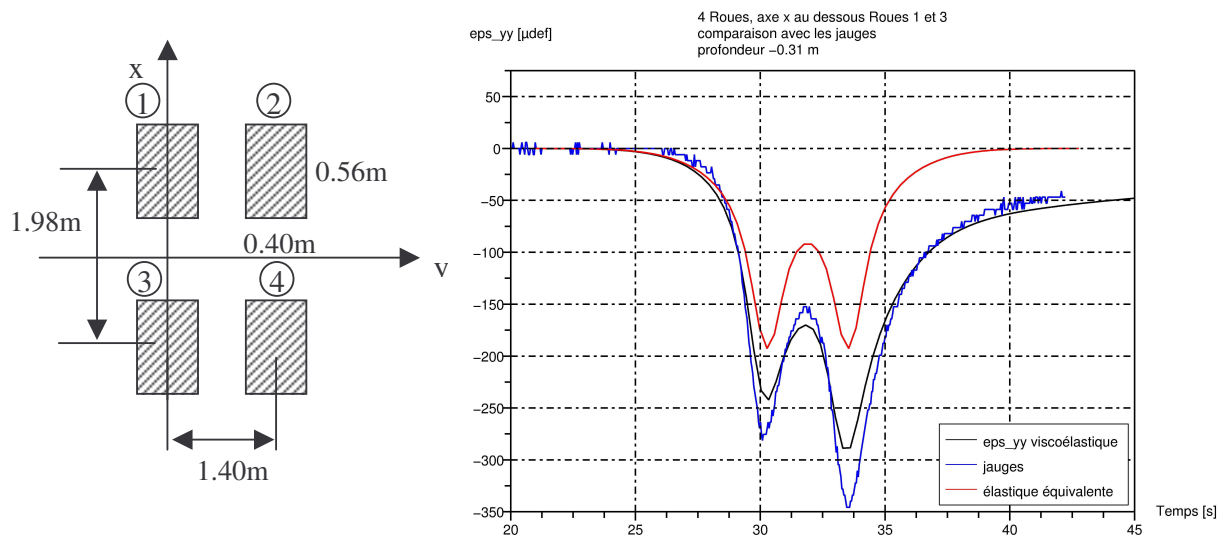


Figure 6 – Comparison between elastic calculus, ViscoRoute calculus and transversal strain measured at the bottom of the bituminous layers under a moving 4-wheels loading (Loft, 2005)

In that modelling, several assumptions on mechanical material properties have been made : similar visco-elastic properties of BB and GB layers, elastic assumptions for other layers, inverse elastic determination on the Young moduli for the soil, precise location of strain sensors. Although, the elastic classical model appears unsuited to obtain a realistic description of the strains measured at the bottom of the bituminous layer, in particular the bending strains under the tires which control the possible fatigue damage of the material. In fact, particularly in that multiple loading case, transversal strain are maximum. Furthermore, due to the visco-elastic delay, there is an accumulation of strain at the end of the second moving load that can not be predicted by an elastic model. As for FEM calculus made with CVCR module of CESAR-LCPC (Petitjean *et al.*, 2002), ViscoRoute visco-elastic results indicate clearly that the visco-elasticity of bituminous materials needs to be taken into account to have a more realistic simulation of the strains produced by the slow loading aircraft flexible pavement.

#### 4 VISCO-ELASTICITY RESULTS COMPARED WITH THE FRENCH DESIGN METHOD

In that section, on five structures made with different thickness bituminous layer, we examine an assumption of the French design method. This assumption consists in taking into account the visco-elastic behaviour with its equivalent elastic modulus at the  $15^{\circ}\text{C}$  French average temperature and the 10 Hz frequency that supposed to be equivalent to the standard 72 km/h French vehicle speed. Then maximum strains at the bottom layer need to be calculated and compared with fatigue experimental results.

The loading chosen in that study is the single French standard moving 65kN uniform load on the squared surface with sides  $2a=2b=0.22158 \text{ m}$ . These structures are composed such as:

- Structure 1: 0.06m GB thickness layer ( $\nu=0.35$ ,  $E_0 = 70MPa$ ,  $E_\infty = 35000Mpa$ ,  $h=0.62$ ,  $k = 0.19$ ,  $\delta = 2.9$ ,  $A_0 = 1.377, A_1 = -0.35698$ ,  $A_2 = 0.001743$ ), 0.24m GNT (unbound graded material) ( $\nu=0.35$ ,  $E=400$  MPa) layered on the soil 1 ( $\nu=0.35$ ,  $E=50$  MPa),
- Structure 2: 0.08m GB thickness layer, 0.24 m GNT on the soil 1,
- Structure 3: 0.12m GB thickness layer, 0.24 m GNT on the soil 1,
- Structure 4: 0.08 m BB thickness layer ( $\nu=0.35$ ,  $E_0 = 50MPa$ ,  $E_\infty = 31000Mpa$ ,  $h=0.49$ ,  $k = 0.16$ ,  $\delta = 2.6$ ,  $A_0 = 3.29747, A_1 = -0.38598$ ,  $A_2 = 0.001827$ ), 0.20m GB thickness layer, 0.35 m GNT on the soil 1,
- Structure 5: 0.08 m BB thickness layer, 0.35m GB thickness layer on the soil 2 ( $\nu=0.35$ ,  $E=120$  MPa).

For all the five structure, comparisons between elastic calculus made with the equivalent elastic moduli (15°C, 10Hz) and ViscoRoute calculus made with (15°C, 72km/h and several values of the speed of the load) have been made. Results show that the time-frequency assumption is correct for structure 1, 2 and 3 but is not true for thick flexible structures. For instance on structure 5 (Figure 7), the ViscoRoute calculus shows that the value of the speed of the load corresponding to have almost the same maximum bending elastic strain (dashing curve) needs to be higher (216km/h). Furthermore, as shown before, visco-elastic longitudinal strain curves show the delay that elastic calculus can not predict.

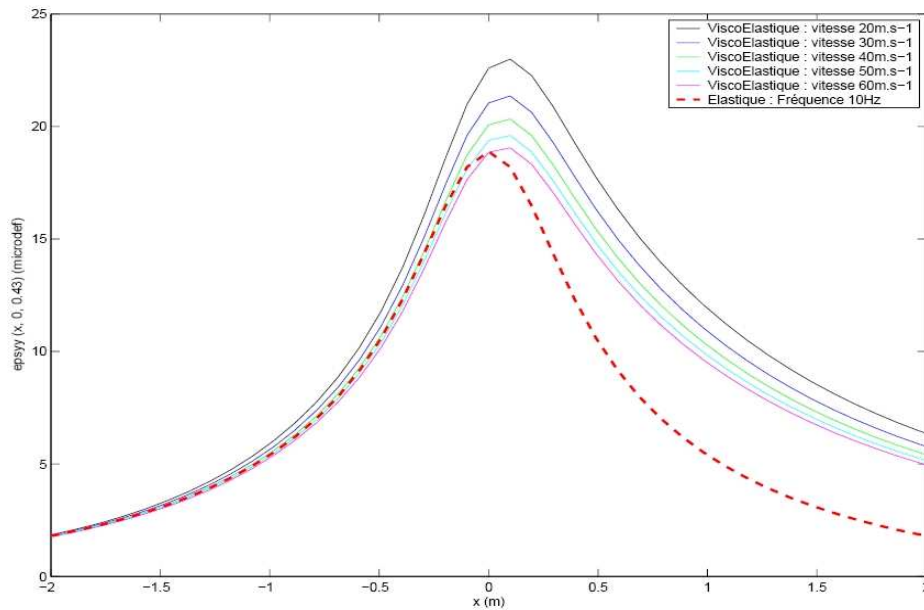


Figure 7 – Transversal strain comparison between Viscoroute calculus (15°C, 72 to 216km/h) and a classical elastic equivalent calculus (15°C, 10Hz) - structure 5 (Poché, 2005)

At last, for the five structures, the influence of the temperature have also been studied. For these bounded layer cases and high temperature such as 40°C, especially for very thin bituminous layer (as structure 1 even as structure 2), results show that the maximum bending strain at the bottom of the thin layer is not the good design criteria to take into account (Loft, 2005).

## 5 CONCLUSION

A thermo-visco-elastic multi-layered model and a new software called ViscoRoute using the Huet-Sayegh behaviour has been developed. The results have been successfully validated by comparison with analytical solutions for infinite half-spaces, with finite element results in a multilayered case and with results from the A380 Pavement experimental program. The assumption of the French design method which takes into account an equivalent elastic modulus at 15°C and 10Hz is investigated. This study confirms that this equivalent elastic modulus and the time-frequency equivalence assumption can be used only for base courses of medium thickness. For bituminous wearing courses and thick flexible pavement structures, especially for aircraft structure and low traffic, it is necessary to develop other concepts.

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